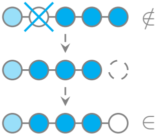
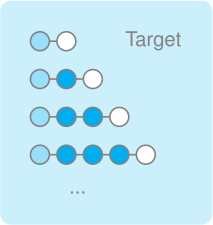
What do you do
if a computational object fails a specification?





Over finite words and regular specifications:

1. Non-deterministic finite automata

2. Deterministic finite automata

“Regular repair of specifications”, in LICS 2011.

What do you do
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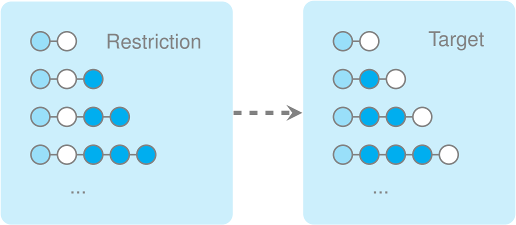


Over finite words and regular specifications:

1. Non-deterministic finite automata
2. Deterministic finite automata

"Regular repair of specifications", in LICS 2011.

At what rate do we need to repair each word in the worst case?

Asymptotic cost

$$\lim_{n\to\infty} \sup \left\{ \underbrace{\frac{\mathrm{cost}(w, T)}{|w|}}_{\text{normalized cost}} \;\middle|\; w \in R,\ |w| \geq n \right\}$$

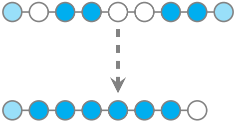
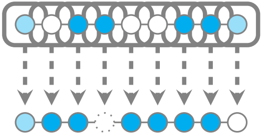
Example

$R : (aa)^*$		$T : (ab)^*$	$\rightarrow$ 1/2
$(\mathrm{a\ a})^N$	- - - →	$(\mathrm{a\ b})^N$	
$R : a^+ b^+$		$T : a^+ c^+$	$\rightarrow$ 1
$\mathrm{a\ b}^N$	- - - →	$\mathrm{a\ c}^N$	

At what rate do we need to repair each word in the worst case?

Asymptotic cost

Different ways of repairing:

Arbitrary	Streaming
	

At what rate do we need to repair each word in the worst case?

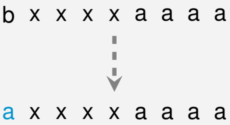
Asymptotic cost

Example

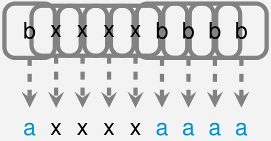
$R: \quad (a + b)\, x^* \,(a^* + b^*)$

$T: \quad a\, x^*\, a^* + b\, x^*\, b^*$

Arbitrary $\to 0$



Streaming $\to 1$



We study how to compute the asymptotic cost of two regular languages

1. The asymptotic cost is rational and can be effectively computed.
2. First algorithm to compute the asymptotic cost:
   - Distance automata and their determinization.
   - Cycle analysis.
3. Streaming asymptotic cost.
   - Reduction to mean-payoff games.
   - Complexity in PTIME.

We study how to compute the asymptotic cost of two regular languages

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The cost of traveling between languages

Cristian Riveros
Michael Benedikt
Gabriele Puppis

University of Oxford
ICALP 2011

Outline

Setting

Edit distance automata

Determinization

Asymptotic Cost

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Setting

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Repairability over regular languages

- Σ and Δ are alphabets.
- Two regular languages:
  - R (Restriction) over Σ^*, and
  - T (Target) over Δ^*.
- R and T are given by:
  - Deterministic finite automata (DFA), or
  - Non-deterministic finite automata (NFA).

In this talk, all automata are trim.

Repairability using edit operations

Edit operations: deletion, insertion, and relabeling.

- All operations have cost equal to 1.

Definition

For words u, v and language T:

$$\mathrm{dist}(u, v) \quad = \quad \text{shortest sequence of operations that transform } u \text{ into } v$$

$$\mathrm{dist}(u, T) \quad = \quad \min_{v \in T} \{ \mathrm{dist}(u, v) \}$$

Both computable in PTIME

(Wagner and Fisher 1974, Wagner 1974).

Asymptotic cost

Definition

$$\mathbf{A}(R, T) = \lim_{n \to \infty} \sup \{ \underbrace{\frac{\mathrm{dist}(w, T)}{|w|}}_{\text{normalized cost}} \mid w \in R, \ |w| \geq n \}$$

Example

$R : (aa)^*$ — $T : (ab)^*$

$(\mathrm{a}\ \mathrm{a})^N \dashrightarrow (\mathrm{a}\ \mathrm{b})^N$

$\mathbf{A}(R, T) = 1/2$

$R : a^+ b^+$ — $T : a^+ c^+$

$\mathrm{a}\ \mathrm{b}^N \dashrightarrow \mathrm{a}\ \mathrm{c}^N$

$\mathbf{A}(R, T) = 1$

Asymptotic cost

Definition

$$\mathbf{A}(R, T) = \lim_{n \to \infty} \sup \left\{ \underbrace{\frac{\mathrm{dist}(w, T)}{|w|}}_{\text{normalized cost}} \;\middle|\; w \in R,\ |w| \geq n \right\}$$

The asymptotic cost $\mathbf{A}(R, T)$ of regular languages R and T:

- always exists.
- is between 0 and 1.

In this talk:

We focus on how to compute $\mathbf{A}(\Sigma^*, T)$ where $R = \Sigma^*$.

Outline

Distance automata

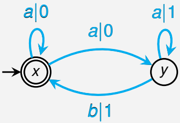
Definition

A distance automaton $\mathcal{D}$ is a non-deterministic finite automaton with transitions labeled with cost in $\mathbb{N} \cup \{\infty\}$.

$$\mathcal{D} : \ \Sigma^* \to \mathbb{N} \cup \{\infty\}$$

Example

$\mathcal{D}$:



$w \ = \ \text{a a b}$

$\rho_1 : \ x \xrightarrow{a|0} y \xrightarrow{a|1} y \xrightarrow{b|1} x \qquad \text{cost}(\rho_1) = 2$

$\rho_2 : \ x \xrightarrow{a|0} x \xrightarrow{a|0} y \xrightarrow{b|1} x \qquad \text{cost}(\rho_2) = 1$

$$\mathcal{D}(w) \ = \ \min\{ \ \text{cost}(\rho_1), \text{cost}(\rho_2) \ \} \ = \ 1$$

$$\mathcal{D}(w) \ = \ \min \{ \ \text{cost}(\rho) \ \mid \ \rho \text{ is a run of } w \text{ over } \mathcal{D} \ \}$$

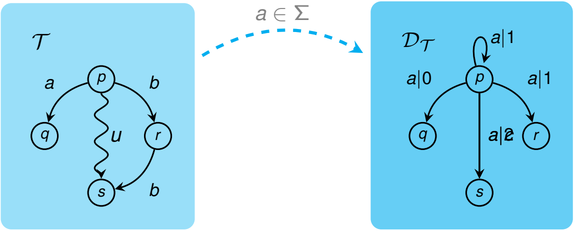
The edit-distance to a language can be computed by a distance automaton

Let $\mathcal{T} = (\Delta, Q, \delta, q_0, F)$ be a finite automaton.

Definition

We define the edit distance automaton $\mathcal{D}_{\mathcal{T}} = (\Sigma, Q, \delta^{\text{edit}}, q_0^{\text{edit}}, F^{\text{edit}})$:

$$\mathcal{D}_{\mathcal{T}}(w) = \text{dist}(w, \mathcal{T}) \quad \text{for all } w \in \Sigma^*$$



$$c = \min_{u \in \Sigma^*} \{ \text{dist}(a, u) \mid p \xrightarrow{u} s \text{ in } \mathcal{T} \}$$

The edit-distance to a language can be computed by a distance automaton

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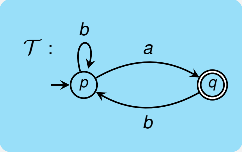
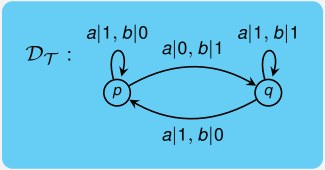
Example





The asymptotic cost problem for a distance automaton

For any distance automaton $\mathcal{D}$:

$$\mathbf{A}(\mathcal{D}) = \lim_{n \to \infty} \sup \left\{ \frac{\mathcal{D}(w)}{|w|} \;\middle|\; w \in \mathcal{L}(\mathcal{D}),\ |w| \geq n \right\}$$

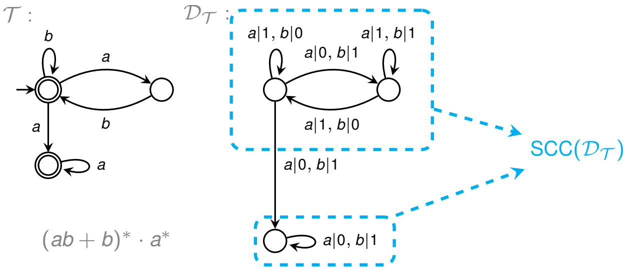
Theorem

The problem of deciding whether $\mathbf{A}(\mathcal{D}) \leq \frac{1}{2}$ is undecidable given an arbitrary distance automaton $\mathcal{D}$.

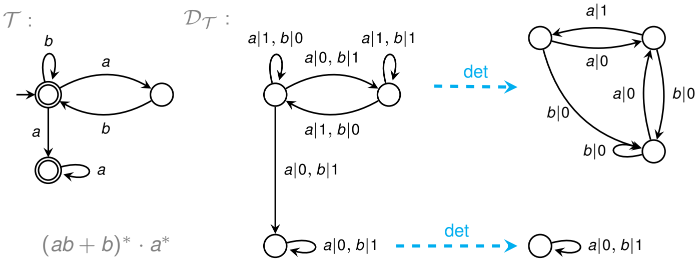
This is not the case for the edit distance automaton $\mathcal{D}_{\mathcal{T}}$.

Outline

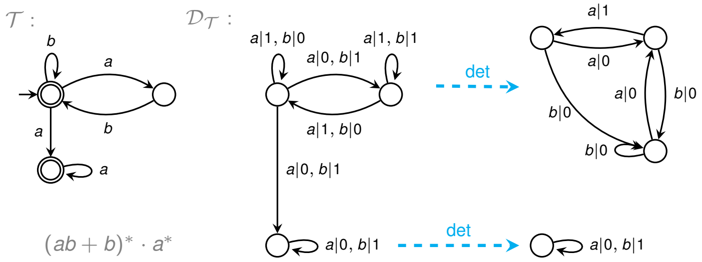
The strongly connected components (SCC) of $\mathcal{D}_{\mathcal{T}}$ are determinizable



The strongly connected components (SCC) of $\mathcal{D}_{\mathcal{T}}$ are determinizable

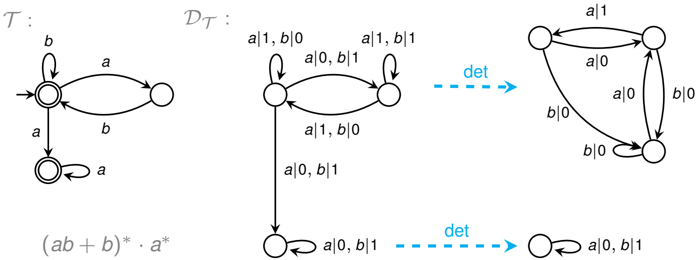


The strongly connected components (SCC) of $\mathcal{D}_\mathcal{T}$ are determinizable



$\mathcal{D}_\mathcal{T}$ is NOT always determinizable.

The strongly connected components (SCC) of $\mathcal{D}_\mathcal{T}$ are determinizable



$(ab + b)^* \cdot a^*$

Proposition

$\mathcal{D}_\mathcal{T}|C$ is determinizable for every $C \in \mathrm{SCC}(\mathcal{D}_\mathcal{T})$.

We can determinize $\mathcal{D}_\mathcal{T}|C$ using Mohri's procedure (Mohri 1997).

The asymptotic cost can be computed using the determinization of a distance automata

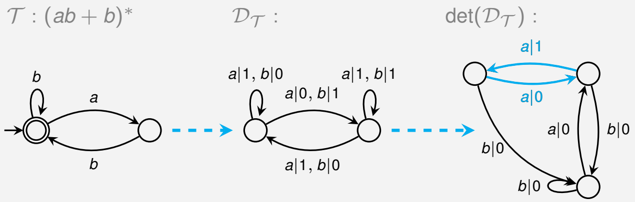
Proposition

Suppose that $\mathcal{D}_{\mathcal{T}}$ is a single strongly connected component:

$$\mathbf{A}(\mathcal{D}_{\mathcal{T}}) = \max\left\{ \frac{\text{cost}(L)}{|L|} \mid L \text{ is a simple cycle of } \det(\mathcal{D}_{\mathcal{T}}) \right\}$$

- $\text{cost}(L) =$ sum of the cost of the edges of L.

Example



Outline

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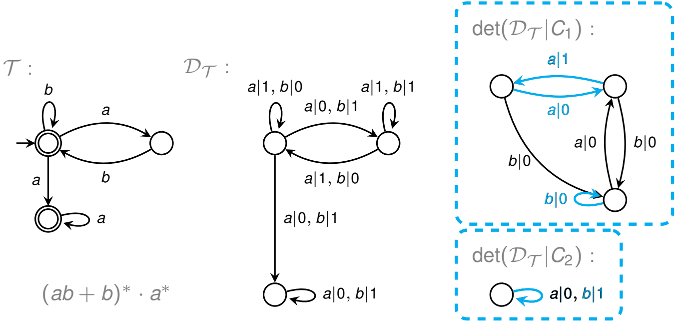
Asymptotic Cost

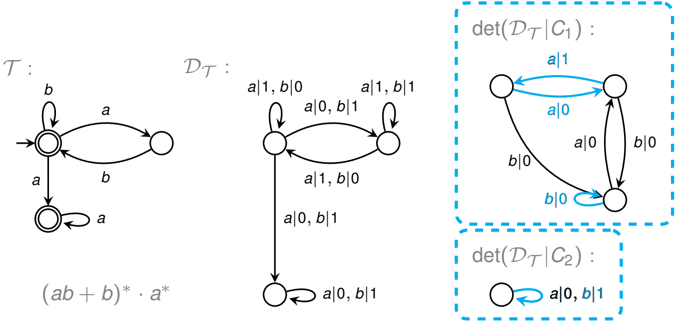
Moving to multiple components



Worst case for C_1: $(aa)\,(aa)\,(aa)\ \ldots$

- $\mathbf{A}((aa)^n, C_1) = \frac{1}{2}$.
- $\mathbf{A}((aa)^n, C_2) = 0$.

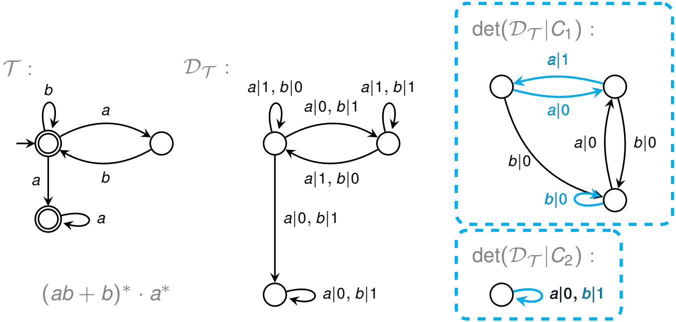
Moving to multiple components



Worst case for C_2: $b\ b\ b\ b\ \ldots$

- $\mathbf{A}(b^n, C_2) \ = \ 1.$
- $\mathbf{A}(b^n, C_1) \ = \ 0.$

Moving to multiple components



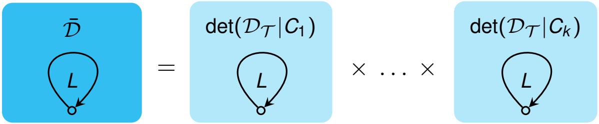
A($\mathcal{D}_\mathcal{T}$) cannot be computed as a function of the simple cycles of det($\mathcal{D}_\mathcal{T}|C$) for all $C \in \mathcal{D}_\mathcal{T}$.

We have to consider a combination of the common cycles of all components

Let $C_1, \ldots, C_k$ be the SCC of $\mathcal{D}_{\mathcal{T}}$.

Definition

We define the multi-distance automaton $\bar{\mathcal{D}}$:



Let $L_1, \ldots, L_m$ be all the simple cycles of $\bar{\mathcal{D}}$.

Definition

$\text{cost}(L_i, C) \quad = \quad$ cost of the projection of the simple cycle L_i into the component $\det(\mathcal{D}_{\mathcal{T}}|C)$ of $\bar{\mathcal{D}}$.

$\mathbf{A}(\mathcal{D}_{\mathcal{T}})$ is equal to a linear combination of its cycles

Theorem

$$\mathbf{A}(\mathcal{D}_{\mathcal{T}}) = \max_{r_1,\ldots,r_m \geq 0} \min_{C \in \mathrm{SCC}(\mathcal{T})} \frac{\sum_{i=1}^{m} r_i \cdot \mathrm{cost}(L_i, C)}{\sum_{i=1}^{m} r_i \cdot |L_i|}$$

- m: number of cycles in $\bar{\mathcal{D}}$.
- $r_1, \ldots, r_m$: number of repetitions of simple cycles $L_1, \ldots, L_m$.

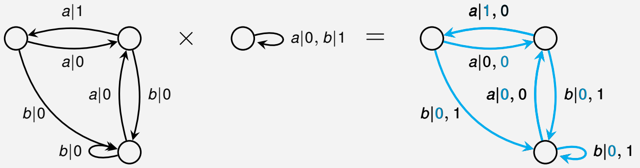
$\mathbf{A}(\mathcal{D}_{\mathcal{T}})$ is equal to a linear combination of its cycles

Theorem

$$\mathbf{A}(\mathcal{D}_{\mathcal{T}}) \;=\; \max_{r_1,\ldots,r_m \geq 0} \;\; \min_{C \in \mathrm{SCC}(\mathcal{T})} \; \frac{\sum_{i=1}^{m} r_i \cdot \mathrm{cost}(L_i, C)}{\sum_{i=1}^{m} r_i \cdot |L_i|}$$

Example $[\ \mathcal{T} :\ (ab + b)^* \cdot a^*\ ]$

$\det(\mathcal{D}_{\mathcal{T}} | C_1)$: $\qquad$ $\det(\mathcal{D}_{\mathcal{T}} | C_2)$: $\qquad$ $\bar{\mathcal{D}}$:



$$\mathbf{A}(\Sigma^*, \mathcal{T}) = \frac{1}{3}$$

Some remarks about computing $\mathbf{A}(\mathcal{D}_{\mathcal{T}})$

$$\mathbf{A}(\mathcal{D}_{\mathcal{T}}) = \max_{r_1,\ldots,r_m \geq 0} \; \min_{C \in \mathrm{SCC}(\mathcal{T})} \frac{\sum_{i=1}^{m} r_i \cdot \mathrm{cost}(L_i, C)}{\sum_{i=1}^{m} r_i \cdot |L_i|}$$

linear programming problem

$$\textbf{MAXIMIZE} \quad y \quad \textbf{SUBJECT TO} \quad \sum_{i=1}^{m} \frac{\mathrm{cost}(L_i, C)}{|L_i|} \cdot x_i \geq y \quad \forall\, C \in \mathrm{SCC}(\mathcal{T})$$

$$\sum_{i=1}^{m} x_i \leq 1$$

$\mathbf{A}(\mathcal{D}_{\mathcal{T}})$ is a rational number.

Some remarks about the complexity of computing $\mathbf{A}(\mathcal{D}_\mathcal{T})$

$\mathbf{A}(\mathcal{D}_\mathcal{T})$ can be computed in double exponential time.

- The size of the multiple-distance automata $\bar{\mathcal{D}}$ is exponential in $\mathcal{D}_\mathcal{T}$.
- The number m of simple cycles is exponential in $\bar{\mathcal{D}}$.
- $\mathbf{A}(\mathcal{D}_\mathcal{T})$ can be reduced to a linear programming problem of size double exponential.

The exact complexity of computing $\mathbf{A}(\mathcal{D}_\mathcal{T})$ is an open problem.

Conclusions and current work

1. The asymptotic cost is rational and can be effectively computed.
2. First algorithm to compute the asymptotic cost:
   - Distance automata and their determinization.
   - Cycle analysis.
3. Streaming asymptotic cost.
   - Reduction to mean-payoff games.
   - Complexity in PTIME.
4. Current work:
   - Repairing tree regular languages.

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